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CALCULATIONS OF THE FLOW OF NATURAL GAS THROUGH CRITICAL FLOW NOZZLES

by Robert C. Johnson Lewis Research Center Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at Winter Annual Meeting of the American Society of Mechanical Engineers Los Angeles, California, November 16-21, 1969



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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

The mass flow rate of methane and nineteen natural gas mixtures through critical flow nozzles has been calculated. The calculation assumes the flow to be one-dimensional and isentropic. The pressure range is 0 to 1000 pounds per square inch and the temperature range is from 450 to 700 degrees Rankine.

From a study of the results, a simple empirical method for making this mass flow rate calculation is proposed. This method would apply to natural gas mixtures whose composition is known and whose components have no more than four carbon atoms.

INTRODUCTION

When critical-flow nozzles are used for metering the mass flow rate of natural gas, the conventional isentropic flow relations do not apply. These equations only apply to a perfect gas. A perfect gas is defined as one having a compressibility factor of unity and an invariant specific heat.

Gases such as air and nitrogen closely approximate this ideal condition at room temperatures and pressures up to a few atmospheres. Natural gas, on the other hand, cannot be considered perfect even at pressures less than atmospheric because natural gas has an appreciable specific-heat variation with temperature. At higher pressures, the compressibility-factor variation also becomes important.

A number of methods have been used to calculate the isentropic mass flow rate of nonperfect gases. One of these methods is that used in reference 1. In this reference, an isentropic exponent is calculated, using the thermodynamic properties of a real gas. This exponent is then used in the conventional critical-flow equation to calculate the mass flow rate. Reference 2, using the tabulated data of reference 3, presents tables of compressible flow functions for the one-dimensional flow of air. Reference 4, using the state equations of reference 3, presents tables of a critical flow factor for air, nitrogen, oxygen, normal hydrogen, parahydrogen, and steam that permit the calculation of the isentropic mass flow rate of these gases through critical-flow nozzles. Reference 4 refers to a number of other reports that describe other methods for making this type of calculation. Reference 5 presents the critical flow factor for nitrogen and helium, and, in addition, describes a computer routine for making these real-gas computations.

In this report, the critical flow factors for methane and two typical natural-gas mixtures are presented. In addition, from a study of nineteen natural gases, an empirical method for calculating both the mass flow rate and upstream volume flow rate of natural gas through critical-flow nozzles is given. This method requires knowledge of the upstream pressure, temperature, and composition of the natural gas. These calculations encompass a temperature range of 450°R to 700°R and a pressure range of 0 to 1000 psia.

NOMENCLATURE

- A Constant in state equation (equation 13)
- ${\bf A_{t.}}$ Area of nozzle throat
- a Constant in state equation (equation 13)

```
Constant in equation 23
ac
           Constant in equation 24
\mathbf{a}_{\mathbf{z}}
           Constant in state equation (equation 13)
ъ
b<sub>c</sub>
           Constant in equation 23
\mathbf{b}_{\mathbf{z}}
           Constant in equation 24
Во
           Constant in state equation (equation 13)
Co
           Constant in state equation (equation 13)
cď
           Discharge coefficient
^{\rm C}_{
m v}
           Specific heat at constant volume
           Ideal-gas specific heat at constant volume
C*
           Critical-flow factor as defined in equation 16
C* perf.
           Perfect-gas critical-flow factor
           Constant in state equation (equation 13)
c
f
           Composition factor as defined in equation 22
G
           Mass flow rate per unit area
Η
           Enthalpy (energy per unit mass)
М
           Molecular weight
m
           Mass flow rate
           Specific-heat ratio
n
           Number of components in mixture
N
           Pressure
p
R
           Specific gas constant
           Gas constant for calculating mass flow rate (equation 20)
R_{m}
           Gas constant for calculating volume flow rate (equation 21)
R,
S
           Entropy
\mathbf{T}
           Temperature
U
           Velocity
```

Volume flow rate

X Mole fraction of component in gas mixture

Z Compressibility factor α Constant in state equation (equation 13) $\alpha_{\mathbf{t}}$ Speed of sound in nozzle throat γ Constant in state equation (equation 13) $\beta_{\mathbf{0}} \cdot \cdot \beta_{7}$ Constants in ideal-gas specific-heat equation (equation 15) ρ Density

Subscripts

p Plenum

t Nozzle throat

i,j Species

k Running index in equation (15)

ANALYSIS

In calculating the mass flow rate of a gas through a critical-flow nozzle, certain assumptions are made. One is that the flow is isentropic from the plenum, where the gas is at rest, to the nozzle throat, where the flow velocity is sonic. The other assumption is that the flow is one-dimensional. These assumptions are represented by the following equations:

$$S_{p} - S_{t} = 0 \tag{1}$$

$$U_{t}^{2} = 2(H_{p} - H_{t})$$
 (2)

Since the flow in the nozzle throat is sonic, an additional equation is:

$$U_t^2 = \alpha_t^2 \tag{3}$$

Expressions for equations 1, 2, and 3 in terms of density and temperature will now be developed.

The differential expressions for entropy and enthalpy are given in reference 6 and are:

$$dS = C_{V} \frac{dT}{T} - \frac{1}{\rho^{Z}} \left(\frac{\partial p}{\partial T} \right)_{\rho} d\rho \tag{4}$$

$$dH = TdS + \frac{p}{\rho^2} d\rho + d\left(\frac{p}{\rho}\right)$$
 (5)

In terms of the compressibility factor, Z, equations 4 and 5 become

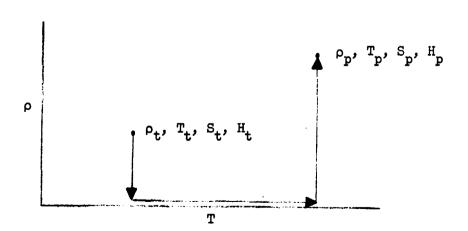
$$\frac{dS}{R} = \frac{C_{V}}{R} \cdot \frac{dT}{T} - \left[Z + T \left(\frac{\partial Z}{\partial T} \right)_{\Omega} \right] \frac{d\rho}{\rho}$$
 (6)

$$\frac{dH}{R} = \frac{C_{V}}{R} dT - T^{2} \left(\frac{\partial Z}{\partial T} \right)_{O} \frac{d\rho}{\rho} + c(ZT)$$
 (7)

where

$$Z = Z(\rho, T) = \frac{p}{\rho RT}$$
 (8)

Equations 6 and 7 are integrated along the path indicated in the following sketch.



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In terms of these integrations, equations 1 and 2 may be written as

$$\frac{S_{p}-S_{t}}{R} = 0 = \int_{T_{t}}^{T_{p}} \frac{C_{v}}{R} \frac{dT}{T} - \ln \frac{\rho_{p}}{\rho_{t}} - \int_{0}^{\rho_{p}} \left[Z - 1 + T \left(\frac{\partial Z}{\partial T} \right)_{\Omega} \right]_{T=T_{p}} \frac{d\rho}{\rho}$$

$$+ \int_{0}^{\rho_{t}} \left[Z - 1 + T \left(\frac{\partial Z}{\partial T} \right)_{\Omega} \right]_{T=T_{t}} \frac{d\rho}{\rho}$$

$$\frac{U_{t}^{2}}{R} = \frac{2(H_{p}-H_{t})}{R} = 2 \left\{ \int_{T_{t}}^{T_{p}} \frac{C_{v}}{R} dT + T_{p} \left[Z_{p} - \int_{0}^{\rho_{p}} \left\{ T \left(\frac{\partial Z}{\partial T} \right)_{\Omega} \right\}_{T=T_{p}} \frac{d\rho}{\rho} \right] \right\}$$

$$- T_{t} \left[Z_{t} - \int_{0}^{\rho_{t}} \left\{ T \left(\frac{\partial Z}{\partial T} \right)_{\Omega} \right\}_{T=T_{t}} \frac{d\rho}{\rho} \right\}$$

$$(10)$$

The term \tilde{C}_v is the ideal-gas specific heat, and represents the specific heat the gas would have in the limit of zero density.

Reference 7 gives an expression for the speed of sound in terms of the compressibility factor. This can be used in equation 3 to yield:

$$\frac{\mathbf{U}_{\mathbf{t}}^{2}}{\mathbf{R}} = \mathbf{T}_{\mathbf{t}} \left\{ \mathbf{Z} + \rho \left(\frac{\partial \mathbf{Z}}{\partial \rho} \right)_{\mathbf{T}} + \frac{\left[\mathbf{Z} + \mathbf{T} \left(\frac{\partial \mathbf{Z}}{\partial \mathbf{T}} \right)_{\rho} \right]^{2}}{\mathbf{C}_{\mathbf{v}}} - \int_{0}^{\rho} \left[\mathbf{Z} \mathbf{T} \left(\frac{\partial \mathbf{Z}}{\partial \mathbf{T}} \right)_{\rho} + \mathbf{T}^{2} \left(\frac{\partial^{2} \mathbf{Z}}{\partial \mathbf{T}^{2}} \right)_{\rho} \right] \frac{d\rho}{\rho} \right\}_{\mathbf{T} = \mathbf{T}_{\mathbf{t}}}^{\mathbf{T}}$$

$$= \mathbf{T}_{\mathbf{t}}$$

$$= \mathbf{T}_{\mathbf{t}}$$

1

In the calculation of the critical mass flow rate, the independent variables would be the plenum temperature and pressure. Equations 9, 10, and 11 involve plenum temperature and density. For this reason, equation 8 has to be solved implicitly for the plenum density. Once the plenum density is known, equations 9, 10, and 11 can be solved for the nozzle throat density, temperature, and velocity. The iteration procedures used in solving these equations are the same as those used in reference 5. Once the gas properties in the nozzle throat are known, the mass flow rate per unit area can be calculated by the following equation.

$$G_{t} = \rho_{t} U_{t}$$
 (12)

BASIC EQUATIONS

One of the basic equations used in this calculation is the state equation that gives the compressibility factor as a function of density and temperature. For a one-component gas, there are many forms of this equation that can be chosen. Natural gas, however, is a mixture of many components. Furthermore, the composition of natural gas varies with time and region. It would therefore be convenient to be able to determine the coefficients of the state equation for a gas mixture from the coefficients of individual components. Such an equation is the one developed by Benedict, Webb, and Rubin in reference 8. This equation is as follows:

$$Z = 1 + (B_0 - \frac{A_0}{RT} - \frac{C_0}{RT}) \rho + (b - \frac{a}{RT}) \rho^2 + \frac{a\alpha}{RT} \rho^5 + \frac{c}{RT3} \rho^2 (1 + \gamma \rho^2) \epsilon^{-\gamma \rho^2}$$
(13)

The coefficients in this equation are determined from the coefficients in the individual mixture components in the following manner (ref. 9).

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$$A_o = \left[\sum_{i=1}^{N} X_i A_{o,i}^{1/2}\right]^2$$

$$B_o = 1/8 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \left[B_{o,i}^{1/3} + B_{o,j}^{1/3} \right]^3$$

$$C_o = \left[\sum_{i=1}^{N} x_i C_{o,i}^{1/2}\right]^2$$

$$z = \left[\sum_{i=1}^{N} x_i a_i^{1/3}\right]^3$$

$$b = \left[\sum_{i=1}^{N} x_i b_i^{1/3}\right]^3$$

$$c = \left[\sum_{i=1}^{N} x_i c_i^{1/3}\right]^3$$

$$\alpha = \left[\sum_{i=1}^{N} x_i \alpha_i^{1/3}\right]^3$$

$$\Upsilon = \left[\sum_{i=1}^{N} X_i \gamma_i^{1/2}\right]^2$$

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Natural gas was assumed to consist of a mixture of seven components. These are methane, ethane, propane, 2-methyl propane, butane, nitrogen, and carbon dioxide. For pipeline gas, it would be unusual to find any appreciable amounts of hydrocarbons heavier than butane. The Benedict-Webb-Rubin coefficients for these gases are listed in Table I.

Table I
Benedict-Webb-Rubin Coefficients For
Components of Natural Gas

	Methane M=16.043	Ethane N=30.070	Propane M=44.097	2-Methyl Propane M=58.124	Butane M=58.124	Nitrogen M=28.016	Carbon Dioxide M=44.011
A _o x 10 ⁻⁴	.699525	1.56707	2.59154	3.85874	3.80296	.449695	1.03904
3 ₀	.682401	1.00554	1.55884	2.20329	1.99211	.733661	•790477
C ₀ x 10 ⁻⁸	2.75763	21.9427	62.0993	103.847	121.305	• 7 19534	16.2657
a x 10 ⁻³	2.98412	20.8502	57.248	117.047	113.705	. 900068	3.94505
Ъ	.867325	2.85393	5•77355	10,889	10.2636	.508467	.596615
c x 10 ⁻⁸	4.98106	64.1314	252.478	559•777	619.256	1.07267	24.6137
α	.511172	1.00044	2.49577	4,41496	4.52693	1.19838	3.23662
r	1.53961	3.0279	5.64525	8.72447	8.72447	1.92451	4.10563

The above coefficients are for temperature in degrees Rankine and density in pound moles per cubic foot. The universal gas constant is taken as 10.7314ft.³ psi per OR per pound mole. The coefficients for all the gases except nitrogen and carbon dioxide are from reference 8. Those for nitrogen are from reference 10 and those for carbon dioxide were determined from a curve fit of the compressibility data in reference 3.

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Another basic equation gives the ideal-gas specific heat as a function of temperature. For a multicompound mixture, this specific heat is given by

$$\frac{\tilde{C}_{v}}{R} = \sum_{i=1}^{N} X_{i} \left(\frac{\tilde{C}_{v,i}}{R_{i}} \right)$$
 (14)

where the ideal-gas specific heat of an individual component is given by

$$\frac{\mathcal{C}_{\mathbf{v}}}{\mathbf{R}} = \beta_{\mathbf{o}} + \sum_{k=1}^{7} \beta_{k} \mathbf{T}^{k} \tag{15}$$

The values of β_k are given in Table II for the seven gases.

Table II

Constants in the Specific-Heat
Polynomial for Components of Natural Gas

—	Polynomial for Components of Natural Gas								
	Methane	Ethane	Propane	Propane	Butane	Nitrogen	Carbon Dioxide		
βο	2.79983	-9.85338	-16.7968	-3.06092	-1.0068	2.50115	2.50447		
β	2.38055 x 10 ⁻³	1.09209 x 10 ⁻¹	1.61581 x 10 ⁻¹	3•37849 x 10 ⁻²	2.5609 x 10 ⁻²	-5.40032 x 10 ⁻⁵	-2.82532 x 10-3		
β2	-8.49323 x 10 ⁻⁶	-3.14401 x 10 ⁻⁴	-4.26262 x 10 ⁻⁴	-1.83299 x 10 ⁻⁵	-7.2622 x 10 ⁻⁶	3.1977 x 10 ⁻⁷	1.49392 x 10 ⁻⁵		
β ₃	4.42759 x 10 ⁻⁹	3.13228 x 10-7	3.8063 x 10 ⁻⁷	2.30647 x 10-9	8.35966 x 10 ⁻¹⁰	-7.60847 x 10 ⁻¹⁰	-6.39673 x 10 ⁻⁹		
β ₁₄	2.30203 x 10 ⁻¹¹	2.3469 x 10-10	3.48189 x 10 ⁻¹⁰	1.02665 x 10 ⁻¹¹	0	6.50205 x 10 ⁻¹³	-2.40307 x 10-11		
β ₅	-1.33172 x 10 ⁻¹⁴	-6.3615 x 10-13	-8.11086 x 10-13	-6.97299 x 10 ⁻¹⁵	0	0	3.2495 x 10 ⁻¹⁴		
β ₆	-2.42459 x 10 ⁻¹⁷	3.17753 x 10-16	3.81236 x 10 ⁻¹⁶	0	0	0	-1.21034 x 10 ⁻¹⁷		
β ₇	1.88221 x 10-20	0	0	0	0	0	0		

1

These coefficients give the correct value of the ideal-gas specific heat over a temperature range of 360°R to 720°R. In all cases, these coefficients were determined from a curve fit of tabulated data. The methane data are from reference 12. These data represent a detailed analysis of spectroscopic data and agree with the data of the American Petroleum Institute (Reference 13) to within one percent. The ethane and propane data below 490°R are from reference 14. The ethane and propane data above 490°R are from reference 13 as are the 2-methyl propane and butane data. The data for nitrogen are from reference 15 and the data for carbon dioxide are from reference 3.

RESULTS AND DISCUSSION

The results of these calculations are presented in terms of a dimensionless critical-flow factor which is defined as follows:

$$C = \frac{G_t \sqrt{RT_p}}{p_p} \tag{16}$$

If the gas were perfect, the expression for the critical-flow factor would only involve the specific-heat ratio and would be

$$C_{perf.}^{*} = \left[n \left(\frac{2}{n+1} \right)^{(n+1)/(n-1)} \right]^{1/2}$$
 (17)

In terms of the critical-flow factor, the actual mass flow rate, $\mathring{\mathbf{m}}$, of a gas through a critical-flow nozzle of geometric throat area, \mathbf{A}_{t} , is given by

$$\dot{m} = C_D A_t C * \frac{P_D}{\sqrt{R_m T_D}}$$
 (18)

The upstream volume flow rate would be given by

$$\dot{V}_{p} = C_{D} A_{t} C * Z_{p} \sqrt{R_{v}^{T_{p}}}$$
(19)

In equations 18 and 19, the discharge coefficient $^{\rm C}_{\rm D}$ mainly represents the effects of the non-isentropic and non-one-dimensional flow in the boundary layer of the nozzle. This coefficient is usually determined by a nozzle calibration and is generally plotted as a function of Reynolds number. Typical values of $^{\rm C}_{\rm D}$ are between 0.96 and 1.0. The results for the nozzle used in reference 16 indicate that $^{\rm C}_{\rm D}$ is independent of Mach number from a value of 0.2, where the flow can be considered incompressible, to a value of 1, where the flow is critical and compressible. Since compressibility effects are negligible, real-gas effects should be negligible because, for the range of temperatures and pressures considered, the real-gas effects can be considered a correction to the compressibility effects.

The specific gas constant in equation 18 is

$$R_{m} = \frac{48.03}{M} \tag{20}$$

where M is the molecular weight. This will give $\mathring{\mathbf{m}}$ in pounds per second if $\mathbf{p}_{\mathbf{p}}$ is in pounds per sq. in., $\mathbf{A}_{\mathbf{t}}$ is in square inches, and $\mathbf{T}_{\mathbf{p}}$ is in degrees Rankine.

The specific gas constant for equation 19 is

$$R_{\mathbf{v}} = \frac{2.398}{M} \tag{21}$$

This will give v_p in cubic feet per second. The units of A_t and T_p are the same as those in equation 20.

The critical-flow factor was calculated for methane and for nineteen natural-gas compositions. These compositions were determined from natural-gas samples taken from pipelines throughout the country. These data were supplied

in a private communication from Mr. Harry Schroyer of the American Meter Company. The range of composition of these gases is listed in Table III.

Any alkane in the natural gas having more than four carbon atoms was ignored. The alkanes that were ignored amounted to less than 0.4 percent by volume; in most cases, the amount was about 0.1 percent by volume.

Figure 1 presents the critical-flow factor for methane. Figures 2 and 3 present the critical-flow factor for two typical natural-gas mixtures. The bulk of the natural gas found in pipelines has a composition similar to the composition of the natural gas of figure 2. The dotted portion of the 450° R curve on figure 3 represents a region in the calculation where the partial pressure of one of the components is above the saturation pressure. It is observed that in the limit of zero pressure, a $1\frac{1}{2}$ percent variation of the critical-flow factor with temperature exists. This is due to the temperature variation of the ideal-gas specific heat. At a given temperature, the variation of the critical-flow factor with pressure is as much as 20 percent for the pressure range of 0 to 1000 pounds per square inch.

Since the composition of natural gas is not fixed, and the critical-flow calculation is complex, it would be desirable to find a convenient empirical method to make this calculation. The method proposed in this paper makes use of a composition factor, f. For methane and the nineteen natural gas mixtures, both the critical-flow factor and the square root of the compressibility are linear functions of this composition factor. This factor is defined as follows:

$$f = X_{C_2H_6} + X_{CO_2} - \frac{1}{2}X_{N_2} + 2X_{C_3H_8} + 3X_{C_4H_{10}}$$
 (22)

The X represents the mole fraction of the component whose chemical symbol appears as a subscript on the X. The accuracy of the critical-flow factor correlation was improved if the correlation was in terms of the product of the critical-flow factor and the square root of the compressibility factor rather than of the critical-flow factor alone. This correlation is represented by the following equation

$$C* \sim Z = a_c f + b_c \tag{23}$$

where a_c and b_c are functions of plenum pressure and temperature and are listed in tables IV and V. In this equation, b_c represents the contribution of the methane and a_c f represents the contribution of the other components. (For a perfect gas with $\gamma = 4/3$, the value of b_c would be 0.6732).

The coefficients in Eq. (22) and the form of the equation are entirely empirical. They result from an intuition regarding the relative contributions of the various components and several trial-and-error assumptions of numerical values of the coefficients. The values calculated by equation 23 agree with those calculated by the methods of this paper to better than 0.1 percent.

In a similar manner, the square root of the compressibility factor can be represented by

$$\sqrt{Z} = a_z f + b_z \tag{24}$$

where a_z and b_z are functions of pressure and temperature and are listed in tables VI and VII. The values calculated by equation 2^{l_1} agree with the values calculated by equation 13 to better than 0.5 percent. In most of the cases, the agreement is better than 0.1 percent.

In terms of the factors calculated by equations 23 and 24, the expressions for mass flow rate and volume flow rate become

$$\dot{\mathbf{m}} = \mathbf{C}_{\mathbf{D}}^{\mathbf{A}}_{\mathbf{t}} \frac{(\mathbf{a}_{\mathbf{c}}^{\mathbf{f}} + \mathbf{b}_{\mathbf{c}})}{(\mathbf{a}_{\mathbf{z}}^{\mathbf{f}} + \mathbf{b}_{\mathbf{z}})} \cdot \frac{\mathbf{p}_{\mathbf{p}}}{\sqrt{\mathbf{R}_{\mathbf{m}}^{\mathbf{T}}_{\mathbf{p}}}}$$
(25)

and $V_p = C_D A_t (a_c f + b_c)(a_z f + b_z) \sqrt{R_v T_p}$ (26)

Since there appears to be no significant loss of accuracy in the use of equations 25 and 26, their use is recommended rather than the more involved calculations described in the analysis section of this paper.

In all the calculations in this paper, a state equation is involved. It should, therefore, be remembered that the accuracy of the results is probably no better than the accuracy of the state equation.

SUMMARY

The mass flow rate of methane and nineteen natural gas mixtures through critical flow nozzles has been calculated. The calculation assumes the flow to be one-dimensional and isentropic. The pressure range is 0 to 1000 pounds per square inch and the temperature range is from 450 to 700 degrees Rankine.

From a study of the results, a simple empirical method for making this mass flow rate calculation is proposed. This method would apply to natural gas mixtures whose composition is known and whose components have no more than four carbon atoms.

TABLE III

Range of Composition
of Nineteen Natural Gases

Gas	Mole Fraction
Methane	0.840 - 0.967
Ethane	0.003 - 0.110
Propane	Trace - 0.020
2-Methyl Propane	Trace - 0.004
Butane	Trace - 0.004
Nitrogen	0.001 - 0.023
Carbon Dioxide	Trace - 0.017

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Value of a

Plenum temperature,				Plems	m pressure,	psia					
deg R	0	100	200	300	400	500	600	700	800	900	1000
450	-0.0265	-0.0297	-0.0330	-0.0365	-0.0399	-0.0430	-0.0452	-0.0458	-0.0435	-0.0361	-0.0206
460	-0.0272	-0.0302	-0.0334	-0.0366	-0.0398	-0.0426	-0.044R	-0.0457	-0.0443	-0.0394	-0.0290
470	-0.0279	-0.0308	-0.0338	-0.0368	-0.0398	-0.0424	-0.0445	-0.0455	-0.0448	-0.0414	-0.0343
480	-0.0285	-0.0313	-0.0342	-0.0371	-0.0398	-0.0423	-0.0442	-0.0453	-0.0451	-0.0428	-0.0376
490	-0.0292	-0.0318	-0.0346	-0.0373	-0.0399	-0.0422	-0.0441	-0.0452	-0.0452	-0.0437	-0.0399
500	-0.0278	-0.0324	-0.0350	-0.0375	-0.0400	-0.0422	-0.0440	-0.0451	-0.0454	-0.0443	-0.0416
510	-C.0304	-0.0329	-0.0353	-0.0378	-0.0401	-0.0422	-0.0439	-0.0451	-0.0455	-0.0448	-0.0427
520	-0.0310	-0.0333	-0.0357	-0.0380	-0.0402	-0.0422	-0.0439	-0.0450	-0.0455	-0.0451	-0.0436
530	-0.0315	-0.0338	-0.0361	-0.0383	-0.0404	-0.0423	-0.0439	-0.0450	-0.0456	-0.0454	-0.0443
540	-0.0321	-0.0343	-0.0365	-0.0386	-0.0406	-0.0424	-0.0439	-0.0450	-0.0456	-0.0456	-0.0448
550	-0.0326	-0.0347	-0.0368	-0.0388	-0.0407	-0.0425	-0.0439	-0.0450	-0.0457	- 3.0458	-0.0452
560	-0.0331	-0.0351	-0.0371	-0.0391	-0.0409	-0.0426	-0.0440	-0.0451	-0.0458	-0.0459	-0.0455
570	-0.0335	-0.0355	-0.0375	-0.0393	-0.0411	-0.0427	-0.0440	-0.0451	-0.0458	-0.0461	-0.0458
590	-0.0340	-0.0359	-0.0378	-0.0396	-0.0412	-0.0428	-0.0441	-0.0451	-0.0458	-0.0462	-0.0460
5 70	-0.0344	-0.0362	-0.0380	-0.0398	-0.0414	-0.0428	-0.0441	-0.0451	-0.0459	-0.0462	-0.0462
600	-0.0348	-0.0366	-0.0383	-0.0460	-0.0415	-0.0429	-0.0441	-0.0451	-0.0459	-0.0463	-0.0463
510	-0.0351	-0.0368	-0.0385	-0.0401	-0.0416	-0.0430	-0.0442	-0.0451	-0.0459	-0.0463	-0.0464
620	-0.0354	-0.0371	-0.0387	-0.0403	-0.0417	-0.0430	-0.0442	-0.0451	-0.0458	-0.0463	-0.0464
630	-0.0357	-0.0373	-0.0389	-0.0404	-0.0418	-0.0430	-0.0442	-0.0451	-0.0458	-0.0462	-0.0464
640	-0.0360	-0.0375	-0.0390	-0.0405	-0.0418	-0.0430	-0.0441	-0.0450	-0.0457	-0.0462	-0.0464
650	-0.0362	-0.0377	-0.0392	-0.0406	-0.0418	-0.0430	-0.0441	-0.0449	-0.0456	-0.0461	-0.0463
660	-0.0363	-0.037R	-0.0392	-0.0406	-0.0418	-0.0430	-0.0440	-0.0448	-0.0455	-0.0460	-0.0462
570	-0.0365	-0.0379	-0.0393	-0.0406	-0.041R	-0.0429	-0.0439	-0.0447	-0.0453	-0.0458	-0.0461
6.90	-0.0366	-0.0380	-0.0393	-0.0406	-0.0417	-0.0428	-0.0437	-0.0445	-0.0452	-0.0456	-0.0459
690	-0.0367	-0.0380	-0.0393	-0.0405	-0.0416	-0.0427	-0.0436	-0.0443	-0.0450	-0.0454	-0.0457
700	-0.0367	-0.0380	-0.0392	-0.0404	-0.0415	-0.0425	-0.0434	-0.0441	-0.0447	-0.0452	-0.0455

Table V

Value of b

					-c						
Plenum temperature,				Plenus	pressure, 1	osia					
deg R	0	100	200	300	400	500	600	700	800	900	1000
450	0.6719	0.6715	0.6713	0.6712	0.6713	0.6717	0.6723	0.6733	0.6747	0.6767	0.6791
460	0.6717	0.6714	0.6712	0. 6712	0.6713	0.6717	0.6724	0.6734	0.6748	0.6766	0.6789
470	0.6714	0.6712	0.6711	0. 4711	0.6714	0.6718	0.6725	0.6734	0.6747	0.6764	0.6786
480	0.6712	0.6710	0.6710	0.6711	0.6713	0.6718	0.6725	0.6734	0.6747	0.6763	0.6783
490	0.6709	0.6708	0.6708	0.6709	0.6712	0.6717	0.6724	0.6734	0.6746	0.6761	0.6780
500	0.6707	0.6706	0.6706	0.6708	0.6711	0.6716	0.6723	0.6733	0.6745	0.6759	0.6777
510	0.6704	0.6703	0.6704	0.6706	0.6710	0.6715	0.6722	0.6731	0.6743	0.6757	0.6774
520	0.6701	0.6701	0.6702	0.6704	0.6708	0.6714	0.6721	0.6730	0.6741	0.6755	0.6771
530	0.6698	0.6698	0.6699	0.6702	0.6706	0.6712	0.6719	0.6728	0.6739	0.6752	0.6767
540	0.6694	0.6695	0.6697	0.6700	0.6704	0.6709	0.6717	0.6726	0.6736	0.6749	0.6764
550 .	0.6691	0.6692	0.6694	0.6697	0.6701	0.6707	0.6714	0.6723	0.6734	0.6746	0.6760
560	0.6687	0.6689	0.6691	0.6694	0.6699	0.6704	0.6712	0.6720	0.6731	0.6743	0.6756
570	0.6684	0.6685	0.6687	0.6691	0.6696	0.6701	0.6709	0.6717	0.6727	0.6739	0.6753
580	0.6680	0.6681	0.6684	0.6688	0.6692	0.6698	0.6706	0.6714	0.6724	0.6735	0.6748
590	0.6676	0.6678	0.6680	0.6684	0.6689	0.6695	0.6702	0.6711	0.6720	0.6732	0.6744
600	0.6672	C-6674	0.6677	0.6681	0.6686	0.6692	0.6699	0.6707	0.6717	0.6728	0.0740
610	0.6668	0.6670	0.6673	0.6677	0.6682	0.6688	0.6695	0.6703	0.6713	0.6723	0.6735
620	0.6663	0.6666	0.6669	0.6673	0.6678	0.6684	0.6691	0.6699	0.6709	0.6719	0.6731
630	0.6659	0.6662	0.6665	0.6669	0.6674	0.6680	0.6687	0.6695	0.6705	0.6715	0.6726
640	0.6655	0.6657	0.6661	0.6665	0.6670	0.6676	0.6683	0.6691	0.6700	0.6710	0.6721
650	0.6650	0.6653	0.6656	0.6661	0.6666	0.6672	0.6679	0.6687	0.6696	0.6706	0.6717
660	0.6646	0.6649	0.6652	0.6657	0.6662	0.6668	0.6675	0.6683	0.6691	0.6701	0.6712
670	0.6641	0.6644	0.6648	0.6652	0.6658	0.6664	0.6671	0.6678	0.6687	0.6696	C.6707
680	0.6637	0.6640	0.6643	0.6648	0.6653	0.6659	0.6666	0.6674	0.6682	0.6692	0.6702
690	0.6632	0.6635	0.6639	0.6644	0.6649	0.6655	0.6662	0.6669	0.6678	0.6687	0.6697
700	0.6627	0.6631	0.6635	0.6639	0.6644	0.6651	0.6657	0.6665	0.6673	0.6682	0.6692

Table VI

Value of az

Plenum				Plen	num pressure,	psia					
temperature,									Charles Since		
deg R	0	100	200	300	400	500	600	700	800	900	1000
450	0.	-0.0252	-0.0530	-0.0837	-0.1179	-0.1561	-0.1988	-0.2464	-0.2991	-0.3564	-0.4162
460	0.	-0.0234	-0.049C	-0.0770	-0.1078	-0.1417	-0.1790	-0.2199	-0.2644	-0.3118	-0.3610
470	0.	-0.0218	-0.0454	-0.0710	-0.0989	-0.1293	-0.1622	-0.1978	-0.2360	-0.2762	-0.3175
480	0.	-0.0203	-0.0422	-0.0657	-0.0911	-0.1184	-0.1478	-0.1791	-0.2123	-0.2469	-0.2823
490	0.	-0.0190	-0.0393	-0.0610	-0.0842	-0.1089	-0.1353	-0.1631	-0.1923	-0.2225	-0.2532
500	0.	-0.0178	-0.0366	-0.0567	-0.0780	-0.1005	-0.1243	-0.1493	-0.1752	-0.2018	-0.2288
510	0.	-0.0166	-0.0342	-0.0528	-0.0724	-0.0931	-0.1147	-0.1371	-0.1604	-0.1841	-0.2079
520	0.	-0.0156	-0.0321	-0.0493	-0.0675	-0.0864	-0.1061	-0.1265	-0.1474	-0-1687	-0.1900
530	0.	-0.0147	-0.0301	-0.0462	-0.0630	-0.0804	-0.0984	-0.1170	-0.1360	-0.1552	-0.1743
540	0.	-0.0138	-0.0283	-0.0433	-0.0589	-0.0750	-0.0916	-0.1086	-0.1259	-0.1433	-0.1606
550	0.	-0.0130	-0.0266	-0.0406	-0.0551	-0.0701	-0.0854	-0.1010	-0.1169	-0.1327	-0.1485
560	0.	-0.0123	-0.0251	-0.0362	-0.0518	-0.0656	-0.0798	-0.0942	-0.1088	-0.1233	-0.1377
570	0.	-0.0116	-0.0236	-0.0360	-0.0487	-0.0616	-0.0748	-0.0881	-0.1015	-0.1149	-0.1281
580	0.	-0.0110	-0.0223	-0.0339	-0.0458	-0.0579	-0.0701	-0.0825	-0.0949	-0.1072	-0-1194
590	0.	-0.0104	-0.0211	-0.0320	-0.0432	-0.0545	-0.0659	-0.0774	-0.0889	-0.1003	-0.1116
600	0.	-0.0099	-0.0200	-0.0303	-0.0408	-0.0514	-0.0621	-0.0728	-0.0835	-0.0941	-0.1045
610	0.	-0.0094	-0.0190	-0.0287	-0.0385	-0.0485	-0.0585	-0.0685	-0.0785	-0.0863	-0.0980
620	0.	-0.0089	-0.0180	-0.0272	-0.0365	-0.0458	-0.0552	-0.0646	-0.0739	-0.0831	-0.0921
630	0.	-0.0085	-0.0171	-0.0258	-0.0346	-0.0434	-0.0522	-0.0610	-0.0697	-0.7783	-0.0867
640	0.	-0.0081	-0.0162	-0.0245	-0.0328	-0.0411	-0.0494	-0.0577	-0.0658	-0)739	-0.0618
650	0.	-0.0077	-0.0154	-0.0233	-0.0311	-0.0390	-0.0468	-0.0546	-0.0623	-0.0698	-0.0772
660	0.	-0.0073	-0.0147	-0.0221	-0.0296	-0.0370	-0.0444	-0.0517	-0.0589	-0.0660	-0.0730
670	0.	-0.0070	-0.0140	-0.0211	-0.0281	-0.0352	-0.0422	-0.0491	-0.0559	-0.0625	-0.0691
680	0.	-0.0067	-0.0134	-0.0201	-0.0268	-0.0334	-0.0401	-0.0466	-0.0530	-0.0593	-0.0654
690	0.	-0.0064	-0.0128	-0.0192	-0.0255	-0.0318	-0.0381	-0.0443	-0.0503	-0.0563	-0.0621
700	0.	-0.0061	-0.0122	-0.0183	-0.0243	-0.0303	-0.0363	-0.0421	-0.0479	-0.0535	-0.0589

Tabl. VII

Value of b

Plenum temperature,				Plenum	pressure, p	sia					
deg R	0	100	200	300	400	500	600	700	800	900	1000
	•	100	200	300	400	300	-			,,,,	
450	1.0000	0.9891	0.9780	0.9667	0.9552	0.9434	0.9315	0.9195	0.9075	0.8955	0.8837
460	1.0000	0.9899	0.9796	0.9692	0.9585	0.9478	0.9370	0.9261	0.9152	0.9044	0.8938
470	1.0000	0.9906	0.9810	0.9714	0.9616	0.9518	0.9419	0.9320	0.9221	0.9124	0.9028
480	1.0000	0.9912	0.9824	0.9734	0.9644	0.9553	0.9463	0.9373	0.9283	0.9195	0.9109
497	1.0000	0.9918	0.9836	0.9753	0.9669	0.9586	0.9503	0.9420	0.9339	0.9259	0.9181
500	1.0000	0.9924	0.9847	0.9770	0.9693	0.9616	0.9535	0.9464	0.9389	0.9316	0.9245
510	1.0000	0.9929	0.9857	0.9785	0.9714	0.9643	0.9573	0.9503	0.9435	0.9369	0.9304
520	1.0000	0.9933	0.9866	0.9800	0.9734	0.9668	0.9603	0.9539	0.9477	0.9416	0.9357
530	1.0000	0.9937	0.9875	0.9813	0.9752	0.9691	0.9631	0.9572	0.9515	0.9459	0.9406
540	1.0000	0.9941	0.9883	0.9826	0.9768	0.9712	0.9657	0.9603	0.9550	0.9499	0.9450
550	1.0000	0.9945	0.9891	0.9837	0.9784	0.9732	0.9681	0.9631	0.9582	0.9535	0.9490
560	1.0000	0.9949	0.9898	0.9848	0.9798	0.9750	0.9702	0.9656	0.9612	0.9569	0.9527
570	1.0000	0.9952	0.9904	0.9857	0.9811	0.9766	0.9723	0.9680	0.9639	0.9599	0.9562
580	1.0000	0.9955	0.9910	0.9867	0.9824	0.9782	0.9741	0.9702	0.9664	0.9628	0.9593
590	1.0000	0.9958	0.9916	0.9875	0.9835	0.9797	0.9759	0.9723	0.9688	0.9654	0.9622
600	1.0000	0.9960	0.9921	0.9883	0.9846	0.9810	0.9775	0.9741	0.9709	0.9678	0.9649
610	1.0000	0.9963	0.9926	0.9891	0.9856	0.9823	0.9790	0.9759	0.9729	0.9701	0.9674
620	1.0000	0.9965	0.9931	0.9898	0.9865	0.9834	0.9804	0.9776	0.9748	0.9722	0.9698
630	1.0000	0.9967	0.9935	0.9904	0.9874	0.9845	0.9817	0.9791	0.9766	0.9742	0.9719
640	1.0000	0.9969	0.9939	0.9910	0.9882	0.9856	0.9830	0.9805	0.9782	0.9760	0.9740
650	1.0000	0.9971	0.9943	0.9916	0.9890	0.9865	0.9841	0.9819	0.9797	0.9777	0.9758
660	1.0000	0.9973	0.9947	0.9922	0.9897	0.9874	0.9852	0.9831	0.9812	0.9793	0.9776
670	1.0000	0.9975	0.9950	0.9927	0.9904	0.9883	0.9862	0.9843	0.9825	0.9808	0.9793
680	1.0000	0.9976	0.9953	0.9931	0.9911	0.9891	0.9872	0.9854	0.9838	0.9822	0.9808
690	1.0000	0.9978	0.9956	0.9936	0.9917	0.9898	0.9881	0.9865	0.9849	0.9835	0.9823
700	1.0000	0.9979	0.9959	0.9940	0.9922	0.9905	0.9889	0.9874	0.9861	0.9845	0.9836

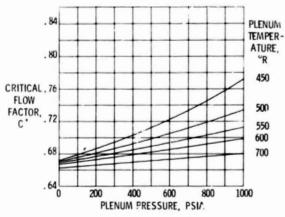


Figure 1. - Critical flow factor for methane.

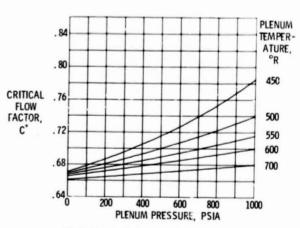


Figure 2. - Critical flow factor for a natural gas. Composition by volume is: 0.9535 methane, 0.0296 ethane, 0.0046 propane, 0.0007 2-methyl propane, 0.0006 butane, 0.0040 nitrogen, and 0.0070 carbon dioxide.

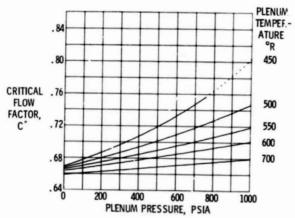


Figure 3. - Critical flow factor for a natural gas. Composition by volume is: 0.8850 methane, 0.0795 ethane, 0.0110 propane, 0.0007 2-methyl propane, 0.0017 butane, 0.0221 nitrogen, and 0 carbon dioxide.